TRINITY COLLEGE

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

SOLUTIONS

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (51 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

A sequence is defined by $T_{n+1} = T_n - 7,\ T_1 = 50$.

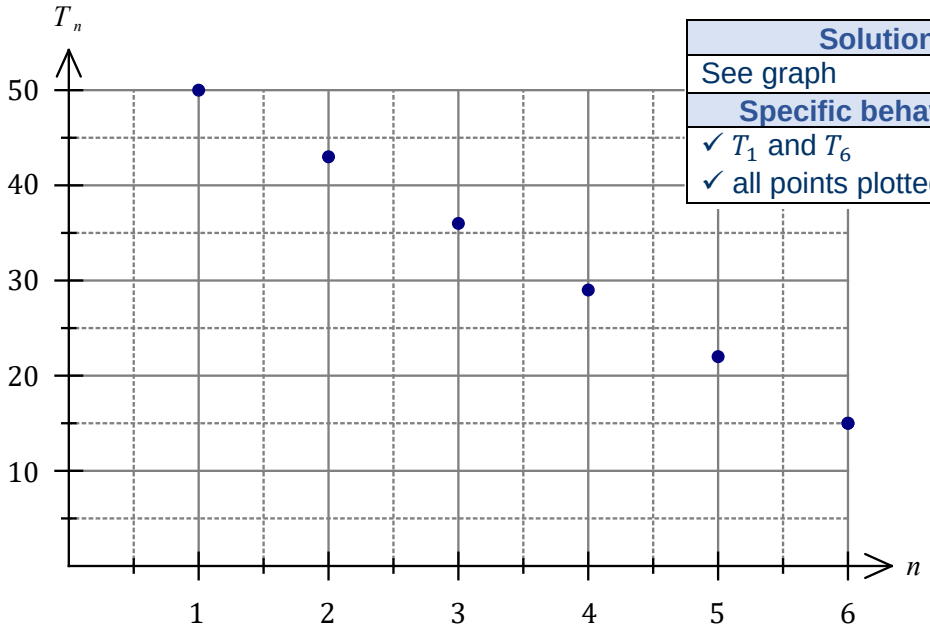
(a) Write the first six terms of the sequence in the following table. (2 marks)

n	1	2	3	4	5	6
T_n	**50**	**43**	**36**	**29**	**22**	**15**

Solution
See table

Specific behaviours
✓ at least 3 correct terms
✓ all correct

(b) Graph the first six terms of the sequence on the axes below. (2 marks)



Solution
See graph

Specific behaviours
✓ T_1 and T_6
✓ all points plotted, no line

(c) The rule for the n^{th} term of the sequence is $T_n = an + b$. Determine the value of the constant a and the value of the constant b. (2 marks)

Solution

$$T_n = 50 + (n-1)(-7)$$
$$= -7n + 57$$

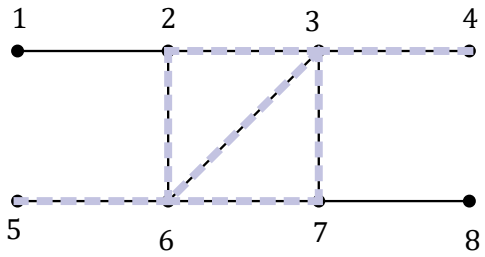
Hence $a = -7$ and $b = 57$.

Specific behaviours
✓ correctly substitutes into n^{th} term rule
✓ simplifies to show value of each constant

See next page

Question 2 **(6 marks)**

Graph G is shown:



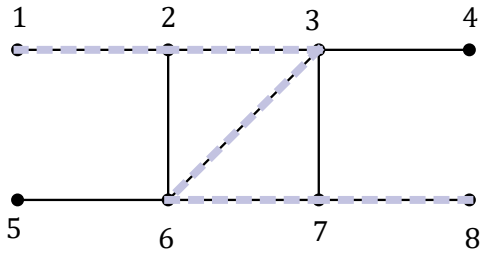
(a) State the number of edges and the number of faces in G and hence show that the graph satisfies Euler's formula. (2 marks)

Solution
$f = 3, \quad e = 9$ $v + f - e = 8 + 3 - 9 = 2$
Specific behaviours
✓ states correct numbers ✓ substitutes and simplifies

(b) State the length of the longest trail in G and highlight the edges in a trail of this length on the graph above. (2 marks)

Solution
Length is 7 edges.
Specific behaviours
✓ highlights any valid trail on graph ✓ correct length

(c) State the length of the longest path in G and highlight the edges in a path of this length on the copy of G below. (2 marks)



Solution
Length is 5 edges.
Specific behaviours
✓ highlights any valid path on graph ✓ correct length

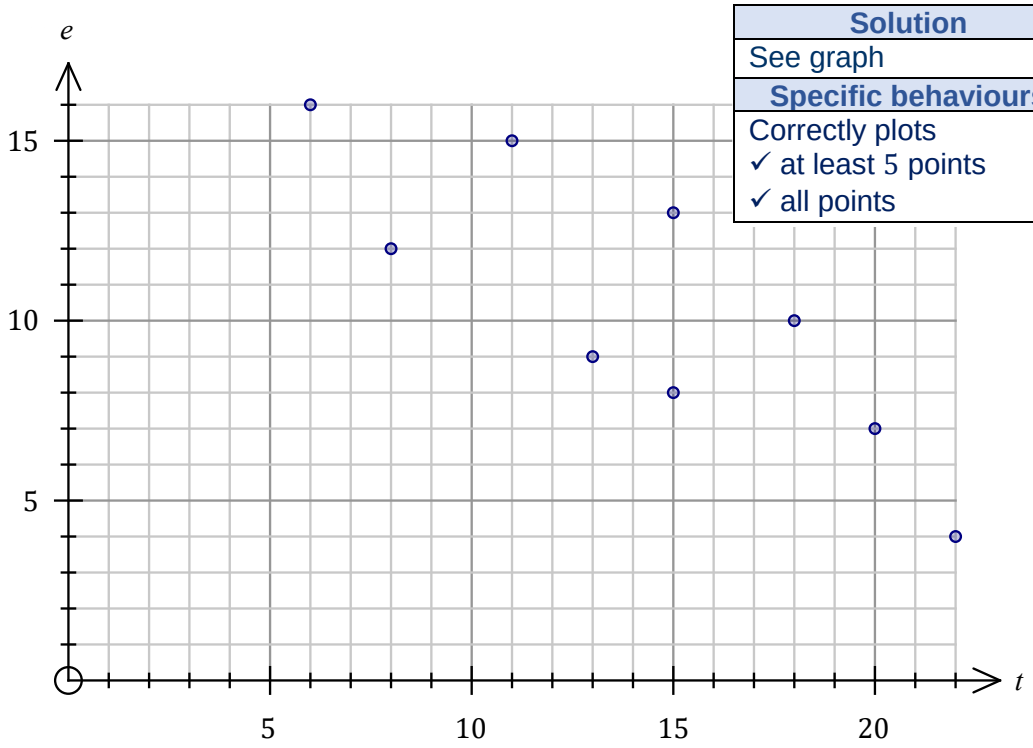
See next page

Question 3 **(6 marks)**

A student recorded the time taken and the number of errors made when completing nine multiple choice tests, each with 40 different questions, in the table below.

Time, t minutes	15	22	20	18	11	15	13	6	8
Number of errors, e	8	4	7	10	15	13	9	16	12

(a) Construct a scatterplot of this data on the axes below. (2 marks)



Solution
See graph
Specific behaviours
Correctly plots ✓ at least 5 points ✓ all points

(b) Describe the strength and direction of the association between the two variables. (2 marks)

Solution
The association is strong and negative.
Specific behaviours
✓ describes strength as moderate or strong ✓ describes direction as negative

(c) The student used the data to conclude that taking more time to answer multiple choice tests caused them to answer more questions correctly. Explain whether this conclusion is justified. (2 marks)

Solution
There is not enough evidence to conclude that there is a causal relationship between the variables. An observed association does not always imply a causal relationship, as there may be other factors involved.
Specific behaviours
✓ indicates causal relationship not justified ✓ states observed association does not always imply causal relationship

See next page

Question 4 **(7 marks)**

(a) Graph G is shown below. Redraw G in the plane, to clearly show that it is planar. (1 mark)

Solution

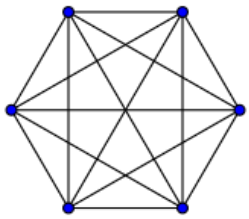
Specific behaviours
✓ correctly drawn in the plane *Penalise once per question for vertices not drawn with solid dot*

(b) Let K_n be the complete graph with n vertices. Draw, and state the number of edges in,

(i) K_2. (2 marks)

Solution
K_2 has 1 edge.
Specific behaviours
✓ correct graph ✓ correct number of edges

(ii) K_6. (2 marks)

Solution
 K_6 has 15 edges.
Specific behaviours
✓ correct graph (all vertices order 5) ✓ correct number of edges

(c) State, with reasoning, whether K_6 is a planar graph. (2 marks)

Solution
K_6 is not a planar graph as it cannot be drawn in the plane. *(Also accept it cannot be drawn so that no two edges cross or only K_1 to K_4 are planar, etc.)*
Specific behaviours
✓ states not planar, with reasoning ✓ sufficient reasoning

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Question 5 **(7 marks)**

Bivariate data analysis of the mass M g, length L mm and width W mm of a large number of snap peas yielded the following correlation coefficients and least-squares lines:

$$r_{WL} = 0.85, \qquad L = 4W + 3, \qquad r_{ML} = 0.7, \qquad L = 15M + 6.$$

(a) Determine the percentage of the variation in the lengths of these snap peas that can be explained by the variation in their masses. (2 marks)

Solution
$r_{ML}^2 = 0.7^2 = 0.49$ 49% of the variation.
Specific behaviours
✓ chooses relevant coefficient ✓ squares and states percentage

(b) One of the least-squares lines would be better than the other as a predictor for the lengths of these snap peas. Write the equation of the line below and explain your choice. (2 marks)

Solution
$L = 4W + 3$ The association between width and length is stronger than between mass and length.
Specific behaviours
✓ chooses and writes relevant line ✓ explains using strength of association

(c) Use the equation from part (b) to predict the length of a snap pea that has a mass of 7 g and a width of 27 mm. (1 mark)

Solution
$L = 4(27) + 3 = 111$ mm
Specific behaviours
✓ calculates length

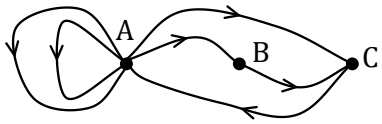
(d) Explain why it is difficult to comment on the validity of the prediction made in part (c). (2 marks)

Solution
Reasonably strong association between the variables supports the validity. However, there is no way of telling if the prediction involves extrapolation, and extrapolation would invalidate the prediction. Hence difficult to comment.
Specific behaviours
✓ indicates strength of association supports validity ✓ indicates no data to check for extrapolation

See next page

Question 6 **(7 marks)**

(a) Digraph G_1 is shown. Complete the adjacency matrix M for G_1. (2 marks)



M	A	B	C
A	**2**	**1**	**1**
B	**0**	**0**	**1**
C	**1**	**0**	**0**

Solution
See matrix
Specific behaviours
✓ one row correct
✓ all rows correct

(b) The adjacency matrix A for the non-directed graph G_2 with 4 vertices is shown below.

$$A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

(i) G_3 is a subgraph of G_2, and has 4 vertices. State, with reasoning, the minimum number of edges that must be removed from G_2 so that G_3 is a simple graph. (3 marks)

Solution
$a_{1,1}$ and $a_{4,4}$ indicate 2 loops. $a_{1,2}$ and $a_{3,4}$ indicate $2 + 1 = 3$ multiple edges. Hence remove $2 + 3 = 5$ edges.
Specific behaviours
✓ identifies loops ✓ identifies multiple edges ✓ correct number of edges

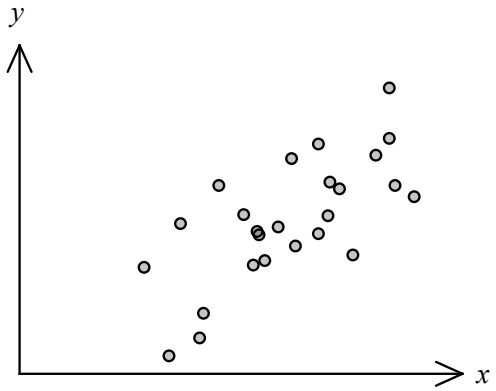
(ii) In the matrix A^4, the entry $a_{3,2} = 68$. Use precise terminology associated with graphs to fully explain the meaning of this entry in A^4. (2 marks)

Solution
G_2 contains 68 different walks of length 4 from vertex f and e.
Specific behaviours
✓ states any two of below ✓ states all three • 68 is the number of different **walks** (NOT paths, ways, etc) • length of each walk is 4 • from f to e

See next page

Question 7 **(6 marks)**

The scatterplot below shows data from 25 samples drawn from different suburbs in a city. The variables are the percentage of people in each sample who have grey hair (x) and who have heart disease (y).



(a) The correlation coefficient r_{xy} for this data is one of $0.7, 0.4, 0.2, -0.2, -0.4, -0.7$.
State r_{xy} and explain your choice. (2 marks)

Solution
$r_{xy} = 0.7$ - the association is positive and moderate.
Specific behaviours
✓ correct value ✓ explains using direction and strength

(b) The least-squares line for the data is $y = ax + b$, where a and b are constants.

(i) State the name of the response variable for this least-squares line. (1 mark)

Solution
Response variable is heart disease, or y.
Specific behaviours
✓ states name or variable

(ii) Explain whether the variable a would be a positive or negative number. (1 mark)

Solution
a would be a positive number as association is positive.
Specific behaviours
✓ states positive with reason

(c) Identify and explain a possible non-causal explanation for the observed association between the variables in this data. (2 marks)

Solution
The age of people in each sample may be a confounding variable. The two variables are more likely to have a causal association with age rather than with each other.
Specific behaviours
✓ states age or another confounding variable ✓ explains common response to age

Alternative Solution
The observed association may be coincidental. Coincidental associations more likely with smaller sample sizes, and here the sample is only 25.
Specific behaviours
✓ states coincidence ✓ suitable explanation

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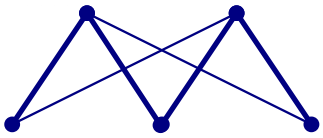
Question 8 **(6 marks)**

(a) Let $K_{m,n}$ be the complete bipartite graph with m vertices in one set and n vertices in the second set. Draw $K_{2,3}$ and explain whether $K_{2,3}$ is Hamiltonian, semi-Hamiltonian or neither. (3 marks)

Solution

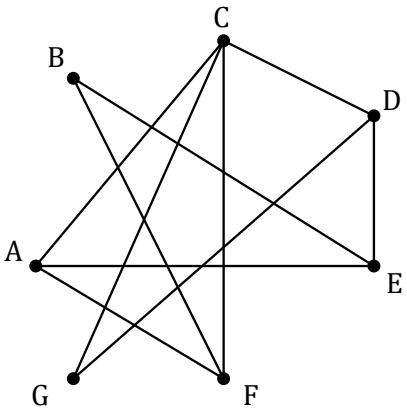


$K_{2,3}$ is semi-Hamiltonian as it contains a Hamiltonian path (example path highlighted) but not a Hamiltonian cycle.

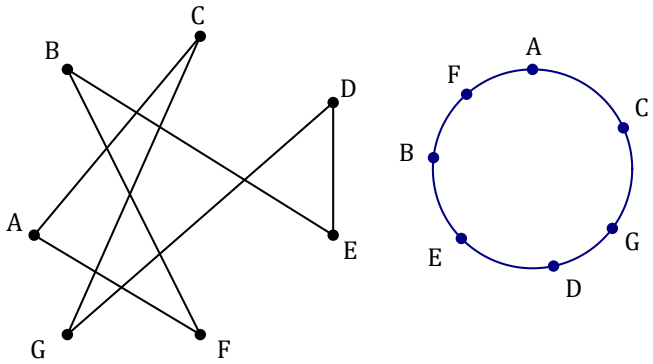
Specific behaviours

✓ draws complete bipartite graph
✓ states semi-Hamiltonian
✓ reasonable explanation

(b) The vertices and edges in the graph below represent students and friendships, respectively. Determine whether it is possible for the students to sit in a circle so that every student is sitting between two friends. If it is possible, draw a possible seating plan. If it is not possible, explain why. (3 marks)



Solution



It is possible as the graph contains a Hamilton cycle.

Specific behaviours

✓ identifies Hamilton cycle
✓ states it is possible
✓ draws correct plan

End of questions

Supplementary page

Question number: _____